Ideal hypothesis testing, and algorithmic information transfer

Bruno Bauwens *

February 27, 2009

In this talk we study influence testing for two discrete time series of equal length, by defining two types of universal influence-free enumerable semimeasures. We prove a coding result that characterizes the semimeasure approximately by a variant of conditional prefix-free Kolmogorov complexity, and describe how influence tests in engineering literature can be considered as approximations of these ideal definitions. We show that these have some nice additive properties. The results shed light on the much more involved case of general ideal hypothesis testing, where many questions are left open.

In statistics a simple hypothesis is defined as a set of logical statements that allow the inference of a unique semimeasure over the set of all a priori possible outcomes of an experiment. Assume that two simple hypothesis have semimeasures P_0 and P_1. The statistical test $d(x) = P_1(x)/P_0(x)$ has an optimal power for any significance level [5]. Composite tests are tests that specify a set S of semimeasures. Multiplicative dominance defines a partial order on the semimeasures, and in some cases the enumerable semimeasures in S have a maximal element, which is called the the universal element of the hypothesis. The likelihood-ratio test can now define in the same way notions of significance and power, which under some assumptions, can have the same meaning as traditional significances and powers.

The hypothesis of a time series x being influence-free from another time series y with equal length $l(x) = l(y) = n$, is a composite hypothesis. We show that with this hypothesis there corresponds a universal enumerable length conditional semimeasure, whose logarithm is approximately

*Department of Electrical Energy, Systems and Automation, Ghent University, Technologiepark 913, B-9052, Ghent, Belgium, Bruno.Bauwens@ugent.be. Supported by a Ph.D grant of the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen). This talk gives an overview of the theoretical section of my PhD work on theoretical and practical independence and influence testing.

given by:

$$K(x|y\uparrow, n) = \min\{l(p) : \Phi(p, n, y_1...y_i) \downarrow = x_{i+1}\},$$

where Φ is a prefix-free Turing machine. This definition also measures the extractable information on the halting problem of y that is transferred to x. In some contexts this might not be appropriate, and more computability restrictions on the causal-free semimeasures can be imposed. A similar coding result will now proved in that case. Due to the standard coding theorem, ideal influence tests against the universal enumerable semimeasure can now be defined as algorithmic and total algorithmic information transfer (IT and TIT), which are analogs of Shannon information transfer [6]. Tests used in engineering practice [4, 7] can be seen as approximations of these ideal influence tests. We also show that IT and TIT define approximate decompositions of algorithmic mutual information. All errors on the above approximate results are of logarithmic order of the algorithmic minimal sufficient statistic of both strings. Generalizations of both encoding and decomposition results of IT and TIT to arbitrary hypothesis tests, lead to many open questions [2].

Identity testing or randomness testing relative to a semimeasure, is testing wether a simple or a composite hypothesis can explain all 'structure' in experimental data. In [3] the existence of such universal independence tests are studied for different computability classes. In [1], some results are shown about the existence of universal identity tests relative to the universal enumerable semimeasure.

References

[1] B. Bauwens. Co-enumerable sumtests for the universal distribution. Submitted, 2008.

[2] B. Bauwens. Ideal hypothesis testing and algorithmic information transfer. In preparation, 2009.

[3] B. Bauwens and S. Terwijn. Notes on sum-tests and independence tests. Submitted, 2008.

[4] C.W.J. Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, 1969.

[5] Neyman J. and Pearson E. On the problem of the most efficient tests of statistical hypotheses. *Philosophical Transactions of the Royal Society of London. Series A*, pages 289–337, 1933.

[6] T. Schreiber. Measuring information transfer. *Physical Review Letters*, 85(2), 2000.

[7] M. Winterhalder, B. Schelter, W. Hesse, K. Schwab, L. Leistritz, R. Bauer, J. Timmer, and H. Witte. Comparison of linear signal processing techniques to infer directed interactions in multivariate neural systems. *Signal Processing*, 85:2137–2160, 2005.